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Effective thermal conductivity of gas–solid composite materials and the temperature difference effect at high temperature

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Abstract

At high temperature the effective thermal conductivity of a bulk material with a large temperature difference may differ from the local equivalent thermal conductivity of a unit cell at the mean temperature due to radiation. This paper analyzed the local equivalent thermal conductivity and the effective thermal conductivity for porous materials with cylindrical and spherical air cavities. Prediction for a spherical case agrees well with experimental data. Investigation shows that the equivalent thermal conductivity at mean temperature can well represent the effective thermal conductivity for a cavity diameter of < 5 mm and porosity $< 78\%$ although the local equivalent conductivity can vary enormously.

Nomenclature

A	surface area
F	view factor
h	half length of a unit cell
H	length of the bulk material
k	thermal conductivity
$\mathbf{n}$	normal vector
Q	heat flux
R	thermal resistance and diameter of cavity
S	cross-sectional area
T	temperature
V	volume
w	porosity.

Greek symbols

Γ	interface boundary
σ	Stefan–Boltzmann constant.

Subscripts

a	top surface temperature of the bulk materials
b	bottom surface temperature of the bulk materials
eff	effective thermal conductivity
eq	local equivalent conductivity
g	gaseous phase
i	ith cell
r	thermal radiation
s	solid phase
Ω	rearrange parts
1	phase 1/surface above the thin slice/top surface of a unit cell
2	phase 2/surface below the thin slice/bottom surface of a unit cell.

1. Introduction

The interest in effective thermal conductivity of multiphase composite materials has continued for over a century. This is essentially due to the wide application of composite materials in industry. The bulk features of the materials depend on factors such as composition, internal structure and external environment. It will be, however, too complicated to consider all the factors in the study of the effective thermal conductivity. No report of universal models incorporating all the influencing factors has yet been found.

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As reflected in the literature, there are still many researchers working in this field in recent years in the hopes of making further progress [1–10]. Their works mainly concentrated on two aspects. The first was to introduce more refined mathematical averaging methods to get an exact or approximate analytical solution. Auriault et al. [1–4] developed the double scale method by means of asymptotic developments. Furmański et al. [5, 6] investigated the effective thermal conductivity of heterogeneous materials by using the ensemble averaging technique. Gu [7] reviewed the Rayleigh method and introduced the transformation field method and the allocation method. The second is to consider the effects of geometry factors such as shape, size, etc. on the effective thermal conductivity. Various parameters were introduced to modify the original equations in order to extend their application range or to improve the accuracy. Allitt et al. [8] compared the accuracy of some models with experimental data and introduced a correction factor, Porosity Shape Factor. Verma et al. [9, 10] considered the influence of factors such as sphericity of the particles, particle cross-section ratio, etc. and introduced some correction factors too.

The research work in this field mainly focused on the effects of composition and internal structure on the effective thermal conductivity of composite materials and the thermal conductivity of the bulk material is usually taken as that of a unit cell at some average temperature. In fact, the temperature distribution over the bulk material may have effects on the effective thermal conductivity, especially for those with large temperature difference. For instance, temperature difference over thermal insulation layers in industrial furnaces can be beyond 1000°C. In this paper, the widely used composite materials, porous materials with air as an enclosed disperse phase is considered. For this kind of material, the environment temperature difference may influence the effective thermal conductivity due to thermal radiation in cavities. The effect of the thermal radiation is considered and the local equivalent thermal conductivity of unit cells is derived as a function of local temperature. Then the effective thermal conductivity for the bulk material is obtained using the local equivalent thermal conductivity as a function of temperature. Finally, comparison is made between the effective thermal conductivity of the bulk material and the equivalent conductivity at mean temperature.

2. The local equivalent conductivity and effective thermal conductivity of the bulk material

Heat transport in porous materials is in three ways: (1) heat conduction in solid and air; (2) thermal radiation between cavity surface; and (3) convection of gas in cavities. These ways are always coupled with each other. Therefore, the heat transfer in porous materials is very complicated. To simplify the problem, the following assumptions are employed in this paper, which were adopted by some researchers [9, 10]. (1) The porous material has a periodic structure and the period is very small compared with the bulk material. (2) The convection in cavities is negligible and (3) cavity surface is assumed to be black and air to be transparent when the effect of thermal radiation in cavities is considered.

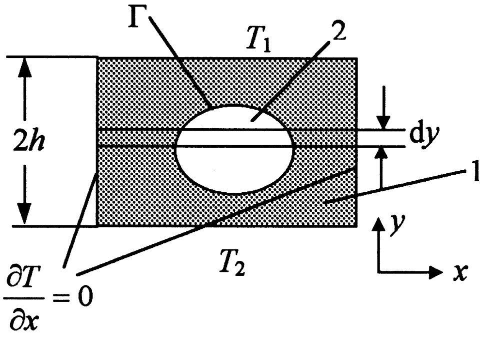

Fig. 1. A unit cell of periodic porous material.

The analysis starts from investigating the properties of one of the periodic unit cells. The results will then be extended to the bulk material. Figure 1 illustrates the geometry of such a unit cell. At steady state, the heat transfer in the cell can be described by

$$\nabla \cdot (k_i \nabla T_i) = 0 \quad i = 1, 2. \tag{1}$$

On the boundary, one has

$$T_1|_\Gamma = T_2|_\Gamma$$

$$k_1 \frac{\partial T_1}{\partial \mathbf{n}}\bigg|_{\mathrm{d}A_1} = k_2 \frac{\partial T_2}{\partial \mathbf{n}}\bigg|_{\mathrm{d}A_1} + \sigma T^4_{\mathrm{d}A_1} - \int_\Gamma \sigma T^4_{\mathrm{d}A_2}\, \mathrm{d}F_{\mathrm{d}A_1-\mathrm{d}A_2}$$

where k is the thermal conductivity, T is the absolute temperature, subscripts 1 and 2 indicate solid and air, respectively, Γ is the boundary between the two phases, $\mathrm{d}A_1$, $\mathrm{d}A_2$ are arbitrary infinitesimal areas on Γ, $\mathbf{n}$ is a normal unit vector of $\mathrm{d}A_1$ and $\mathrm{d}F$ the view factor. Here the cavity surface is assumed to be black.

Although equation (1) has a very simple form, it is almost impossible to obtain an analytical solution because of the complexity of geometry and boundary conditions. To solve the problem, further assumptions are necessary, that is, the temperature linearly distributes along the heat flux over a unit cell. According to the previous assumption, the size of a unit cell is very small compared with the bulk materials. Therefore, this assumption on temperature distribution of a unit cell will not have much influence on the temperature over the bulk material. Verma et al. [10] employed this assumption when they investigated the thermal properties of solid–solid composite materials.

The bulk material can be regarded as a series connection of the unit cells. So the derivation of the equivalent thermal conductivity of the unit cells is the starting point of this work.

Each unit cell consists of solid and gas phases. The unit cell can be divided into thin slices normal to the heat flux with thickness dy as shown in Fig. 1. The thin slices are in two categories: solid slices composed of only solid phase and solid–air slices composed of solid and gas phases. From the definition of thermal conductivity, the local equivalent thermal conductivity of these slices can be determined. For the first category,

$$k_1 = k_s. \tag{2}$$

For the second

$$k_2 = (Q_s + Q_g + Q_r) \bigg/ \left(\frac{dT}{dy} \cdot S\right) \tag{3}$$

where S is the cross-section of the slice, Q_s is the conduction heat flux through the solid part with the cross-section S_s,

$$Q_s = k_s \frac{dT}{dy} \cdot S_s. \tag{4}$$

Q_g is the conduction flux through the air with the cross-section S_g,

$$Q_g = k_g \frac{dT}{dy} \cdot S_g. \tag{5}$$

Q_r is the radiation heat flux through the thin slice and can be expressed by

$$Q_r = \int_{A_1} \int_{A_2} \sigma T^4_{dA_i} \, dF_{dA_i - dA_j} \, dA_i - \int_{A_2} \int_{A_1} \sigma T^4_{dA_j} \, dF_{dA_j - dA_i} \, dA_j \tag{6}$$

where A_1 and A_2 indicate the areas of cavity surface which are above and below the thin slice, respectively, dA_i and dA_j are arbitrary infinitesimal areas on A_1 and A_2, respectively, σ is Stefan–Boltzmann constant. Due to the linear temperature assumption over the cell, the temperature gradient is

$$\frac{dT}{dy} = \frac{T_1 - T_2}{2h} \tag{7}$$

where T_1, T_2 are boundary temperatures of the unit cell and 2h is the length of the unit cell. Combining equations (3)–(6) yields

$$k_2 = k_s \frac{S_s}{S} + k_g \frac{S_g}{S} + \frac{Q_r}{\frac{T_1 - T_2}{2h} \cdot S}. \tag{8}$$

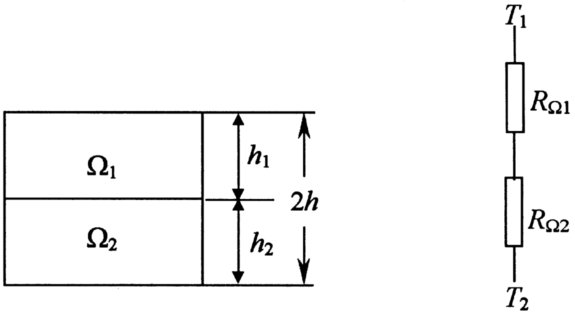

Fig.2a

Fig.2b

Fig. 2. (a) Series connection of two parts Ω_1 and Ω_2. (b) Network of Fig. 2(a).

Suppose all the thin solid slices can be rearranged together as part Ω_1 and all the thin solid–gas slices can be rearranged together as part Ω_2. The unit cell then becomes a series connection of the two parts, as shown in Fig. 2(a). The total thermal resistance of the unit cell is obtainable by adding the thermal resistance of the two parts, as shown in Fig. 2(b),

$$R_{\text{cell}} = R_{\Omega_1} + R_{\Omega_2} \tag{9}$$

where R_{Ω_1} and R_{Ω_2} are the thermal resistance of Ω_1 and Ω_2, respectively and are related to the equivalent thermal conductivity of the two parts. Because of the linear temperature gradient assumption, the average equivalent thermal conductivities of the two parts are used,

$$k_{\Omega_1} = \int_{h_1} k_1 \, dy / h_1 = k_s \tag{10}$$

$$k_{\Omega_2} = \frac{k_s}{h_2 S} \int_{h_2} S_s \, dy + \frac{k_g}{h_2 S} \int_{h_2} S_g \, dy + \frac{1}{\frac{T_1 - T_2}{2h} h_2 S} \int_{h_2} Q_r \, dy. \tag{11}$$

Then

$$R_{\Omega_1} = \frac{h_1}{k_s S} \tag{12}$$

$$R_{\Omega_2} = \frac{h_2^2}{k_s \int_{h_2} S_s \, dy + k_g \int_{h_2} S_g \, dy + \frac{2h}{T_1 - T_2} \int_{h_2} Q_r \, dy}. \tag{13}$$

Using equations (9), (12) and (13), the equivalent thermal conductivity of the unit cell becomes

$$k_{\text{eq}} = \left(\frac{h_1}{2 k_s h} + \frac{h_2^2 S}{2h \left(k_s \int_{h_2} S_s \, dy + k_g \int_{h_2} S_g \, dy + \frac{2h}{T_1 - T_2} \int_{h_2} Q_r \, dy \right)} \right)^{-1}. \tag{14}$$

So far the equivalent thermal conductivity for a local unit cell or a period $k_{\rm eq}$ is obtained. In the case that two phases are solid, the equivalent thermal conductivity of a unit cell can be simply regarded as the effective thermal conductivity of the bulk material due to the absence of thermal radiation. However, for the porous materials, $k_{\rm eq}$ is the function of local temperature because of thermal radiation in cavities and is different from that of the bulk material. Consider the one-dimensional problem as shown in Fig. 3, where H and H_i indicate thickness of the bulk material and the ith unit cell along the heat flux, respectively. $T_{\rm a}$ and $T_{\rm b}$ are the temperatures on the boundary of the bulk material and $k_{\rm eq}(T_i)$ is the equivalent thermal conductivity of the ith unit cell. To guarantee the heat flux conservation along the y-direction, one has

$$\frac{k_{\rm eff}(T_{\rm b}-T_{\rm a})}{H}=\frac{k_{\rm eq}(T_i)\Delta T_i}{H_i}=\frac{\sum_{i=1}^{n}k_{\rm eq}(T_i)\Delta T_i}{\sum_{i=1}^{n}H_i} \tag{15}$$

where $k_{\rm eff}$ is the effective thermal conductivity of the bulk material, ΔT_i is the temperature difference over the ith unit cell. From equation (15), $k_{\rm eff}$ can be written as

$$k_{\rm eff}=\frac{\int_{T_{\rm a}}^{T_{\rm b}}k_{\rm eq}\,{\rm d}T}{T_{\rm b}-T_{\rm a}}. \tag{16}$$

Because the size of the period Ω is very small compared with the bulk size of the porous material, integration is employed here to replace the series summation.

Although the effective thermal conductivity of porous materials can be calculated by combining equation (14) with (16) for irregular cavity shape, it is still very difficult to get an analytical expression due to the existence of thermal radiation. The following paragraphs will present two cases with cylindrical and spherical cavities to see the temperature difference effect.

(1) Case 1: cylindrical cavity

Cavities are assumed to have a cylindrical shape vertical to the heat flux and distributed uniformly in a solid. A unit cell is selected and a polar coordinate system is applied to the unit cell, as shown in Fig. 4. This is a two-dimensional problem. Based on the linear temperature distribution assumption, the temperature of an arbitrary point on the cavity surface can be expressed by

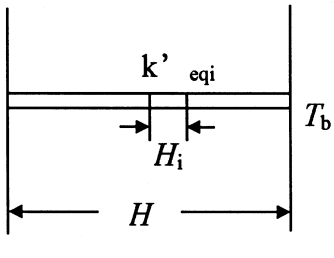

Fig. 3. Illustration of the local unit cell and the bulk material.

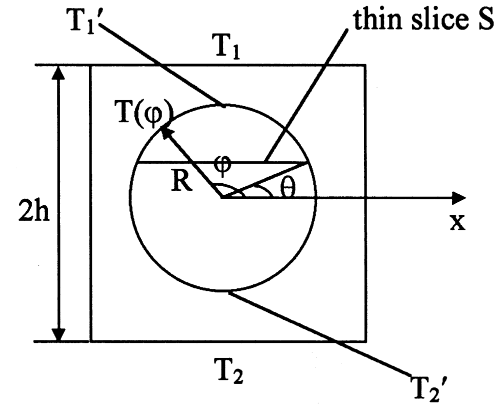

Fig. 4. A unit cell of the porous material with cylindrical cavities.

$$T(\varphi)=a\sin\varphi+b \tag{17}$$

where

$$a=\tfrac{1}{2}(T'_1-T'_2)$$
$$b=\tfrac{1}{2}(T'_1+T'_2) \tag{18}$$

T'_1 and T'_2 are the temperatures of the top and bottom point on the cavity surface, respectively and can be expressed as

$$T'_1=\frac{R}{2h}(T_1-T_2)+\frac{1}{2}(T_1+T_2)=a+b \tag{19}$$

$$T'_2=-\frac{R}{2h}(T_1-T_2)+\frac{1}{2}(T_1+T_2)=-a+b \tag{20}$$

T_1 and T_2 are the temperatures on the top and bottom boundary of the unit cell.

The position of a thin slice is shown in Fig. 4 with $\theta\in[-\pi/2,\pi/2]$. An infinitesimal strip is taken from the cavity surface. If the strip is above the thin slice, that is $\varphi\in[\theta,\pi-\theta]$, the view factor from the infinitesimal strip to the thin slice can be expressed as [11]

$$F_{{\rm d}\varphi-S}=\frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\left(\sin\frac{\varphi}{2}+\cos\frac{\varphi}{2}\right). \tag{21}$$

If the infinitesimal strip is below the thin slice, that is $\varphi\in[\pi-\theta,2\pi+\theta]$, the view factor is

$$F_{{\rm d}\varphi-S}=\frac{\sqrt{2}}{2}\cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\left(\sin\frac{\varphi}{2}-\cos\frac{\varphi}{2}\right). \tag{22}$$

The net radiative heat flow through the thin slice can be expressed by

$$Q_{\rm r}(\theta)=\int_{\theta}^{\pi-\theta}F_{{\rm d}\varphi-S}\sigma T^4(\varphi)R\,{\rm d}\varphi$$

$$
\begin{aligned}
&-\int_{\pi-\theta}^{2\pi+\theta} F_{\mathrm{d}\varphi-S}\sigma T^4(\varphi)R\,\mathrm{d}\varphi \\
&= \sigma R\left[a^4\left(\frac{2}{63}\sin^3\theta\cos\theta+\frac{16}{315}\sin\theta\cos\theta\right)\right. \\
&\quad +a^3b\left(\frac{4}{7}\sin^2\theta\cos\theta+\frac{8}{35}\cos^3\theta\right. \\
&\quad \left.+\frac{24}{35}\cos\theta\right)+a^2b^2\left(\frac{8}{5}\sin\theta\cos\theta\right) \\
&\quad \left.+ab^3\left(\frac{16}{3}\cos\theta\right)\right]. \qquad (23)
\end{aligned}
$$

From equation (23), the integral term in equation (14), $\int_{h_2} Q_{\mathrm{r}}\,\mathrm{d}y$, can be obtained

$$
\begin{aligned}
\int_{h_2} Q_{\mathrm{r}}\,\mathrm{d}y &= \int_{-\pi/2}^{\pi/2} Q_{\mathrm{r}}\,\mathrm{d}(R\sin\theta) \\
&= \sigma R^2\left(\frac{4\pi}{5}a^3b+\frac{8\pi}{3}ab^3\right). \qquad (24)
\end{aligned}
$$

The other terms in equation (16) can also be obtained,

$$
\int_{h_2} S_{\mathrm{s}}\,\mathrm{d}y = \int_{-\pi/2}^{\pi/2} (2h-2R\cos\theta)\,\mathrm{d}(R\sin\theta) = 4hR-\pi R^2 \qquad (25)
$$

$$
\int_{h_2} S_{\mathrm{g}}\,\mathrm{d}y = \int_{-\pi/2}^{\pi/2} 2R\cos\theta\,\mathrm{d}(R\sin\theta) = \pi R^2. \qquad (26)
$$

From equations (24)–(26) and (14), the expression of the equivalent thermal conductivity of the unit cell is

$$
k_{\mathrm{eq}} = \left[\frac{h-R}{k_{\mathrm{s}}h} + \frac{4R}{k_{\mathrm{s}}(4h-\pi R)+k_{\mathrm{g}}\pi R+8\pi\sigma R^2\left(\frac{4}{5}a^2b+\frac{2}{3}b^3\right)}\right]^{-1}. \qquad (27)
$$

By comparing the two terms, $\frac{4}{5}a^2b$ and $\frac{2}{3}b^3$, it is apparent that the first item is much smaller than the second one. Equation (27) can be simplified by neglecting the term $\frac{4}{5}a^2b$,

$$
k_{\mathrm{eq}} = \left[\frac{h-R}{k_{\mathrm{s}}h} + \frac{4R}{k_{\mathrm{s}}(4h-\pi R)+k_{\mathrm{s}}\pi R+\frac{16}{3}\pi\sigma R^2T^3}\right]^{-1} \qquad (28)
$$

where $T = b = (T_1+T_2)/2$. The effective thermal conductivity of the composite material can be obtained by substituting equation (28) into equation (16),

$$
\begin{aligned}
k_{\mathrm{eff}} &= \frac{1}{A} - \frac{1}{3A^2CD^2(T_{\mathrm{b}}-T_{\mathrm{a}})}\ln\frac{T_{\mathrm{b}}+D}{T_{\mathrm{a}}+D} \\
&\quad + \frac{1}{6A^2CD^2(T_{\mathrm{b}}-T_{\mathrm{a}})}\ln\frac{T_{\mathrm{b}}^2-DT_{\mathrm{b}}+D^2}{T_{\mathrm{a}}^2-DT_{\mathrm{a}}+D^2} \\
&\quad - \frac{1}{\sqrt{3}A^2CD^2(T_{\mathrm{b}}-T_{\mathrm{a}})}\left(\mathrm{arctg}\frac{2T_{\mathrm{b}}-D}{\sqrt{3}D}-\mathrm{arctg}\frac{2T_{\mathrm{a}}-D}{\sqrt{3}D}\right) \qquad (29)
\end{aligned}
$$

where coefficients A, B, C and D are

$$
A = \frac{1}{k_{\mathrm{s}}}\left[1-\left(\frac{4w}{\pi}\right)^{1/2}\right] \qquad (30)
$$

$$
B = k_{\mathrm{s}}\left[\left(\frac{\pi}{4w}\right)^{1/2}-\frac{\pi}{4}\right]+\frac{\pi k_{\mathrm{g}}}{4} \qquad (31)
$$

$$
C = \tfrac{4}{3}\pi\sigma R \qquad (32)
$$

$$
D = \left(\frac{B}{C}+\frac{1}{AC}\right)^{1/3} \qquad (33)
$$

w is the porosity of the material

$$
w = \frac{V_{\mathrm{g}}}{V} = \frac{\pi R^2}{4h^2}. \qquad (34)
$$

(2) Case 2: spherical cavity

For a unit cell with spherical cavities, a spherical coordinate system is applied as shown in Fig. 5. The temperature of an arbitrary point on the cavity surface can be expressed as

$$
T(\varphi) = a\cos\varphi+b. \qquad (35)
$$

Coefficients a and b have the same expression as equation (18). The temperatures at the top and bottom points on the cavity surface are T_1' and T_2', respectively and can be calculated from equations (19) and (20). T_1 and T_2 are

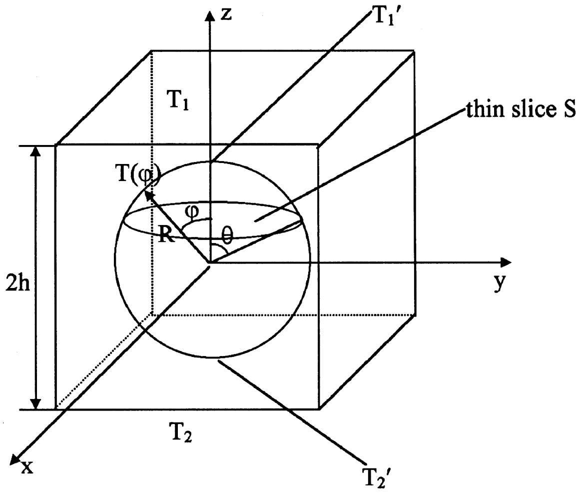

Fig. 5. A unit cell of the porous material with spherical cavities.

the temperatures on the top and bottom boundary of the unit cell, respectively.

The view factor between two arbitrary infinitesimal areas on the spherical cavity surface is [11]

$$\mathrm{d}F_{\mathrm{d}A_1-\mathrm{d}A_2}=\frac{1}{4\pi R^2}\mathrm{d}A_2. \tag{36}$$

The location of a thin slice is shown in Fig. 5 for $\theta\in[0,\pi]$. Here S_1 and S_2 are employed to denote the part of the cavity surface above and below the thin slice, respectively.

Similar to the procedure in the cylindrical cavity case, the equivalent thermal conductivity of a unit cell is

$$k_{\mathrm{eq}}=\left[\frac{h-R}{k_{\mathrm{s}}h}+\frac{8hR}{k_{\mathrm{s}}\left(8h^2-\frac{4}{3}\pi R^2\right)+\frac{4}{3}k_{\mathrm{g}}\pi R^2+\frac{16}{3}\sigma\pi R^3b^3}\right]^{-1} \tag{37}$$

and the effective thermal conductivity of the bulk porous materials with spherical cavities

$$k_{\mathrm{eff}}=\frac{1}{A}-\frac{1}{3A^2CD^2(T_{\mathrm{b}}-T_{\mathrm{a}})}\ln\frac{T_{\mathrm{b}}+D}{T_{\mathrm{a}}+D}+\frac{1}{6A^2CD^2(T_{\mathrm{b}}-T_{\mathrm{a}})}\ln\frac{T_{\mathrm{b}}^2-DT_{\mathrm{b}}+D^2}{T_{\mathrm{a}}^2-DT_{\mathrm{a}}+D^2}-\frac{1}{\sqrt{3}A^2CD^2(T_{\mathrm{b}}-T_{\mathrm{a}})}\left(\mathrm{arctg}\frac{2T_{\mathrm{b}}-D}{\sqrt{3}D}-\mathrm{arctg}\frac{2T_{\mathrm{a}}-D}{\sqrt{3}D}\right) \tag{38}$$

where A, B, C, D are

$$A=\frac{1}{k_{\mathrm{s}}}\left[1-\left(\frac{6w}{\pi}\right)^{1/3}\right] \tag{39}$$

$$B=k_{\mathrm{s}}\left[\left(\frac{\pi}{6w}\right)^{1/3}-\frac{\pi}{6}\left(\frac{6w}{\pi}\right)^{1/3}\right]+\frac{k_{\mathrm{g}}}{6}\left(\frac{6w}{\pi}\right)^{1/3} \tag{40}$$

$$C=\frac{2}{3}\pi\sigma R\left(\frac{6w}{\pi}\right)^{1/3} \tag{41}$$

$$D=\left(\frac{B}{C}+\frac{1}{AC}\right)^{1/3}. \tag{42}$$

The porosity of the material

$$w=\frac{V_{\mathrm{g}}}{V}=\frac{\pi R^3}{6h^3}. \tag{43}$$

3. Results and discussion

If the cavities in material have simple geometry, for example, cylindrical or spherical ones and distribute uniformly in the solid, the local equivalent and effective thermal conductivities are computable directly from equations (28) and (37) and equations (29) and (38). The parameters involved in these equations are geometry parameters R, porosity w, thermal conductivity of the solid phase and air k_{s} and k_{g}, temperatures on the surface of the bulk materials T_{a} and T_{b} and some physical constants π and σ. Once these parameters are known, the effective thermal conductivity of the bulk material can be easily determined.

3.1. Comparison with experiment

In order to check the accuracy of this approach, the calculation values are compared with available experimental data. Frand and Kingery [12] investigated the effective thermal conductivity of porous material by means of an experimental approach. Several specimens were made in the shape of a one-inch cube. Alumina is used as a solid phase and cavities are spherical. The cavities distributed uniformly in the specimens with a diameter of 0.031 cm. By changing the cavity number in the unit volume, the specimens can have a different porosity. Frand and Kingery [12] only gave the average measurement temperature T_{m}, corresponding to $T_{\mathrm{m}}=(T_{\mathrm{a}}+T_{\mathrm{b}})/2$ in this paper. A temperature difference of 30°C is employed in the calculation of equation (38). The thermal conductivity of alumina [12] is given in Table 1. Table 2 lists the comparison between our calculation results from equations (37), (38) and the experimental data. Parameter e is the divergence between the calculation results and the experimental data.

Table 2, demonstrates that the predictions of equation (38) have very high accuracy. The largest divergence is 11.2%. It can also be found that when the temperature and porosity are low, calculation agrees very well with the experimental data. At high temperature and porosity, however, the divergence increases and the predicted values are generally greater than the experimental data. There are several causes that may account for this phenomena. (1) The cavity surface is not black. As the temperature and porosity increase, equation (38) overpredicts the heat transfer by thermal radiation in cavities. (2) Temperature distribution is different from the linear assumption, especially at large pores and high temperature gradients. (3) The temperature difference over the samples in the experiments may be greater than 30°C as assumed in the above calculation. At the same average temperature, greater difference means lower and higher

Table 1
Thermal conductivity of alumina

T_{m} (°C)	200	400	600	800
k_{s} (W m^{-1} K^{-1})	21.16	12.54	8.36	6.79

Table 2
Comparison of predicted effective thermal conductivity [equation (38)] with the experimental data

T_{m}	200°C			400°C			600°C			800°C		
w (%)	k_{exp} (W m^{-1} K^{-1})	k_{cal} (W m^{-1} K^{-1})	e (%)	k_{exp} (W m^{-1} K^{-1})	k_{cal} (W m^{-1} K^{-1})	e (%)	k_{exp} (W m^{-1} K^{-1})	k_{cal} (W m^{-1} K^{-1})	e (%)	k_{exp} (W m^{-1} K^{-1})	k_{cal} (W m^{-1} K^{-1})	e (%)
0.123	18.29	18.35	0.3	10.97	10.88	0.8	7.32	7.26	0.8	5.75	5.90	2.6
0.234	16.20	15.83	2.3	9.41	9.39	0.2	6.27	6.27	0.0	4.81	5.10	6.2
0.300	14.37	14.41	0.3	8.36	8.55	2.3	5.38	5.71	6.1	4.18	4.65	11.2
0.442	11.50	11.57	0.6	6.79	6.87	1.2	4.44	4.60	4.1	3.40	3.74	10.0
0.487	10.45	10.72	2.6	6.01	6.37	6.0	3.92	4.26	8.7	3.14	3.48	10.8

temperatures at the opposite ends of the specimen. The equivalent thermal conductivity of the unit cell is a function of temperature. At lower temperatures, the thermal radiation is reduced and hence the equivalent thermal conductivity.

3.2. The temperature difference effect

The predictions of equivalent thermal conductivities at mean temperatures are made in Table 3. The values are almost the same as that of equation (38), which, in turn, demonstrates that the influence of a small temperature difference over the bulk material is very small. To see the temperature difference effect on the effective thermal conductivity, Table 4 compares equation (38) with a large temperature difference of 1400°C with the experimental data at a mean temperature 800°C. It also lists the values of equivalent thermal conductivities at lowest and highest temperatures. It can be seen that the temperature difference has almost no effect on the conductivity. Since the values of thermal conductivities of alumina and air at the mean temperature are used, only the variation of thermal radiation in the cavities is considered. One can deduce that the thermal radiation in cavities is negligible in the experimental specimens of [12]. Usually, the temperature effects on conductivities are considered in k_{s} and k_{g} and a linear variation of k_{s} and k_{g} with the temperature is a very good approximation. From equation (16), the temperature difference will not alter the effective thermal conductivity for linear variation. Only the variation of thermal radiation may induce a temperature difference

Table 4
Predictions of equation (38) with temperature difference of 1400°C at mean temperature 800°C and prediction of equation (37) at lowest, highest and mean temperature

w (%)	k_{eff} ($T = 800$°C)	k_{eq} ($T = 800$°C)	k^{*}_{eq} ($T = 100$°C)	k^{*}_{eq} ($T = 1500$°C)
0.123	5.90	5.90	5.90	5.92
0.234	5.10	5.11	5.10	5.15
0.300	4.65	4.66	4.64	4.71
0.442	3.75	3.76	3.74	3.83
0.487	3.48	3.50	3.48	3.58

* Thermal conductivities of alumina and air are the values at the mean temperature.

Table 3
Comparison of the local equivalent thermal conductivity [equation (37)] at the mean temperature with experimental data

T_{m}	200°C			400°C			600°C			800°C		
w (%)	k_{exp} (W m^{-1} K^{-1})	k_{cal} (W m^{-1} K^{-1})	e (%)	k_{exp} (W m^{-1} K^{-1})	k_{cal} (W m^{-1} K^{-1})	e (%)	k_{exp} (W m^{-1} K^{-1})	k_{cal} (W m^{-1} K^{-1})	e (%)	k_{exp} (W m^{-1} K^{-1})	k_{cal} (W m^{-1} K^{-1})	e (%)
0.123	18.29	18.35	0.3	10.97	10.88	0.8	7.32	7.26	0.8	5.75	5.90	2.6
0.234	16.20	15.84	2.2	9.41	9.39	0.2	6.27	6.27	0.0	4.81	5.11	6.2
0.300	14.37	14.41	0.3	8.36	8.55	2.3	5.38	5.72	6.3	4.18	4.66	11.5
0.442	11.50	11.58	0.7	6.79	6.88	1.3	4.44	4.61	3.8	3.40	3.76	10.6
0.487	10.45	10.75	2.9	6.01	6.39	6.3	3.92	4.28	9.2	3.14	3.50	11.5

Table 5
Variation of equivalent and effective thermal conductivities with cavity diameter for $T_m = 800$°C and temperature difference = 1400°C

Cavity	$2R$ (mm)	k_{eff} ($T = 800$°C)	k_{eq} ($T = 800$°C)	k_{eq}^* ($T = 100$°C)	k_{eq}^* ($T = 1500$°C)
Spherical ($w = 0.52$)	1.0	3.22	3.36	3.30	3.62
	5.0	3.36	3.66	3.31	4.94
	10.0	3.53	4.02	3.32	6.58
	20.0	3.86	4.75	3.35	9.86
Cylindrical ($w = 0.78$)	1.0	1.68	1.68	1.54	2.20
	5.0	2.52	2.26	1.56	4.83
	10.0	3.56	3.00	1.60	8.12
	20.0	5.64	4.46	1.66	14.7

* Values of thermal conductivities of alumina at mean temperature are used.

effect. So the following part will focus on the conditions where thermal radiation may play an important role.

From equations (28) and (37), it is easy to know that a large cavity diameter, high porosity and high temperature will increase the fraction of thermal radiation. For a spherical cavity, the largest possible porosity is 0.5236. An extreme case with $w = 0.52$ and $T_m = 800$°C is selected. The temperature difference is set to 1400°C. Table 5 lists the variation of equivalent and effective thermal conductivities with a spherical cavity diameter. The difference between the effective and the equivalent thermal conductivity at the mean temperature is not much. The differences are 14%, for $2R = 10$ mm and 23% for $2R = 20$ mm. For most materials in application, the cavity diameter is below 10 mm or even smaller, so the temperature difference is not important and the equivalent thermal conductivity at the mean temperature can well represent the effective thermal conductivity of the bulk materials. The last two columns in Table 5 tabulate the equivalent thermal conductivity at temperatures of 100 and 1500°C in which the conductivities of alumina and air at the mean temperature are used. At a low temperature, the thermal radiation is not important and is negligible. At a high temperature, however, the thermal radiation is very important, especially at a large cavity diameter. The equivalent thermal conductivity can be beyond the thermal conductivities of component solid and air.

More significant temperature difference effects could be observed for a cylindrical cavity case where the higher porosity can be reached. This is also illustrated in Table 5. Comparison with a spherical case shows that large porosity will result in a greater temperature difference effect and the variation of the equivalent thermal conductivity over the bulk material is much more significant. If the diameter is below 5 mm, however, the effect of the temperature difference effect is not very much.

4. Summary

The effect of the temperature difference over the bulk material on the effective thermal conductivity was discussed in this paper. A concept of local equivalent thermal conductivity was proposed and applied to derive the effective thermal conductivity for the bulk material. Analytical expressions for cavities with cylindrical and spherical shapes were given. Comparison with experimental data of the spherical case is satisfactory. Calculation showed that the local equivalent thermal conductivity over the bulk material differs very much. The temperature difference, however, is not significant if the diameter of the cavity is below 5 mm and the porosity is smaller than 78%. When the porosity is higher than 80% and the cavity diameter is greater than 5 mm, a strong temperature difference effect is expected at a high temperature and the temperature difference effect should be considered.

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